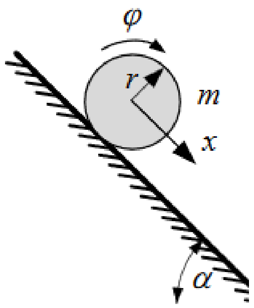
zad.



$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{q}_l}\right) - \frac{\partial E_k}{\partial q_l} + \frac{\partial E_p}{\partial q_l} = 0$$

$$E_k = \frac{I_o \omega^2}{2} + \frac{m v^2}{2} = \frac{m r^2 \left(\frac{\dot{x}}{r}\right)^2}{4} + \frac{m \dot{x}^2}{2} = \frac{3}{4} m \dot{x}^2$$

$$E_p = -m\, g\, x\, sin\varphi$$

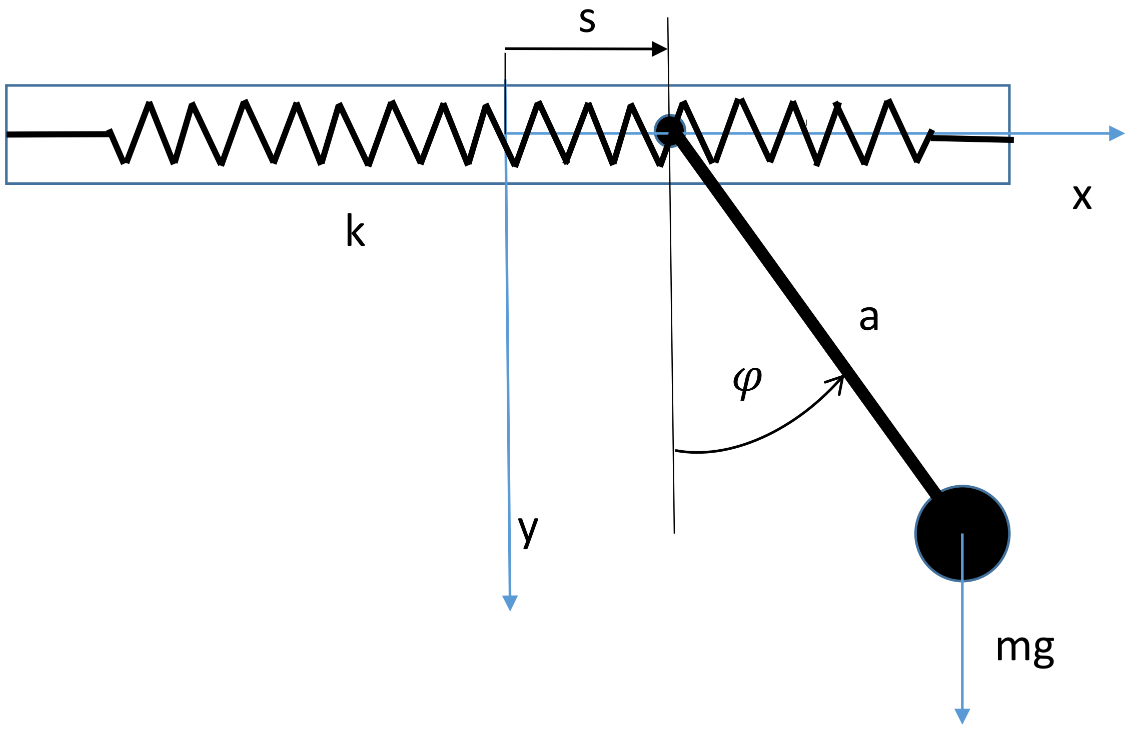
$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{x}}\right) = \frac{d}{dt}\left(\frac{3}{2} m \dot{x}\right) = \frac{3}{2} m \ddot{x}$$

$$\frac{\partial E_k}{\partial x} = 0$$

$$\frac{\partial E_p}{\partial x} = -m\, g\;\; sin\varphi$$

Równanie ruchu: $\frac{3}{2} m \ddot{x}\; - m\;\; g\;\; sin\varphi = 0$ ⇒ $\ddot{x}\; = \frac{2}{3} g\;\; sin\varphi$

Zadanie: Ułożyć równania różniczkowe ruchu wahadła matematycznego



$$x = s + a sin\varphi$$

$$y = a cos\varphi$$

$$\dot{x} = \dot{s} + a cos\varphi \cdot \dot{\varphi}$$

$$\dot{y} = -a sin\varphi \cdot \dot{\varphi}$$

$$E_k = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2)$$

1 trygonometryczna

$$E_k = \frac{1}{2} m(\dot{s}^2 + 2\dot{s} a cos\varphi \cdot \dot{\varphi} + a^2 \dot{\varphi}^{\,2})$$

$$E_p = -mgy + \frac{ks^2}{2} = -mga cos\varphi + \frac{1}{2} ks^2$$

N=2

1 $$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{s}}\right) - \frac{\partial E_k}{\partial s} + \frac{\partial E_p}{\partial s} = 0$$

2 $$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\varphi}}\right) - \frac{\partial E_k}{\partial \varphi} + \frac{\partial E_p}{\partial \varphi} = 0$$

1

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{s}}\right)=\frac{1}{2}m\frac{d}{dt}(2\dot{s}+2a\dot{\varphi}cos\varphi)=m(\ddot{s}+a(\ddot{\varphi}cos\varphi-\dot{\varphi}sin\varphi\cdot\dot{\varphi})$$

$$\frac{\partial E_k}{\partial s}=0 \qquad \frac{\partial E_p}{\partial s}=ks$$

$$\boxed{m\ddot{s}+ma\ddot{\varphi}cos\varphi-ma\dot{\varphi}^2sin\varphi+ks=0}$$

2

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\varphi}}\right)=\frac{1}{2}m\frac{d}{dt}(2a^2\dot{\varphi}+2a\dot{s}cos\varphi)=ma^2\ddot{\varphi}+ma\ddot{s}cos\varphi-ma\dot{s}\cdot sin\varphi\cdot\dot{\varphi}$$

$$\frac{\partial E_k}{\partial \varphi}=-\frac{1}{2}m2a\dot{s}\dot{\varphi}\sin\varphi \qquad \frac{\partial E_p}{\partial \varphi}=mgasin\varphi$$

$$ma^2\ddot{\varphi}+ma\ddot{s}cos\varphi-ma\dot{s}\cdot sin\varphi\cdot\dot{\varphi}+ma\dot{s}\dot{\varphi}\sin\varphi+mgasin\varphi=0$$

$$\boxed{ma^2\ddot{\varphi}+ma\ddot{s}cos\varphi+mgasin\varphi=0}$$

uzyskujemy układ równań

$$m\ddot{s} + ma\ddot{\varphi}cos\varphi - ma\dot{\varphi}^{\,2}sin\varphi + ks = 0$$

$$ma^2\ddot{\varphi} + ma\ddot{s}cos\varphi + mgasin\varphi=0$$

Równania są nieliniowe, więc układ linearyzujemy:

Dla małych drgań $sin\varphi = \varphi, cos\varphi = 1$

Elementy wyższego rzędu, zakładamy, że są równe zero

$$\dot{\varphi}^{\,2}sin\varphi = 0$$

Końcowe równania:

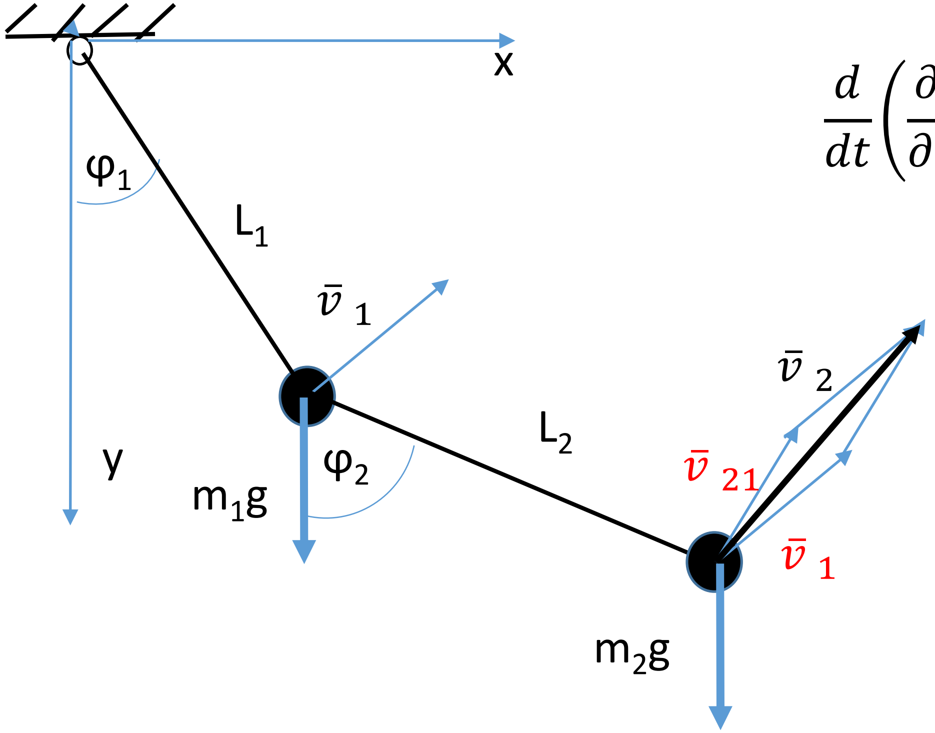
$$m\ddot{s} + ma\ddot{\varphi} + ks = 0$$

$$a\,\ddot{\varphi} + \ddot{s} + g\varphi=0$$

$m \neq 0$

$a \neq 0$

zadanie: wahadło podwójne



$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\varphi}_1}\right) - \frac{\partial E_k}{\partial \varphi_1} + \frac{\partial E_p}{\partial \varphi_1} = 0$$

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\varphi}_2}\right) - \frac{\partial E_k}{\partial \varphi_2} + \frac{\partial E_p}{\partial \varphi_2} = 0$$

$$E_k = \frac{m_1 {v_1}^2}{2} + \frac{m_2 {v_2}^2}{2}$$

$$v_1 = \dot{\varphi}_1 L_1 \qquad v_{21} = \dot{\varphi}_2 L_2$$

suma wektorów $(v_2)^2 = (v_1)^2 + (v_{21})^2 + 2v_1 v_{21}\cos(\varphi_2 - \varphi_1)$

$$E_k = \frac{m_1 {\dot{\varphi}_1}^2 L_1^2}{2} + \frac{m_2}{2}\left({\dot{\varphi}_1}^2 L_1^2 + {\dot{\varphi}_2}^2 L_2^2 + 2\dot{\varphi}_1 L_1 \dot{\varphi}_2 L_2 cos(\varphi_2 - \varphi_1)\right)$$

$$E_p = -m_1 g L_1 cos\varphi_1 - m_2 g(L_1 cos\varphi_1 + L_2 cos\varphi_2)$$

1

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\varphi}_1}\right) - \frac{\partial E_k}{\partial \varphi_1} + \frac{\partial E_p}{\partial \varphi_1} = 0$$

$$E_k = \frac{m_1 {\dot{\varphi}_1}^2 L_1^2}{2} + \frac{m_2}{2}\left({\dot{\varphi}_1}^2 L_1^2 + {\dot{\varphi}_2}^2 L_2^2 + 2\dot{\varphi}_1 L_1 \dot{\varphi}_2 L_2 cos(\varphi_2 - \varphi_1)\right)$$

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\varphi}_1}\right) = \frac{d}{dt}(m_1 \dot{\varphi}_1 L_1^2 + m_2 \dot{\varphi}_1 L_1^2 + m_2 L_1 \dot{\varphi}_2 L_2 cos(\varphi_2 - \varphi_1))$$

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\varphi}_1}\right) = m_1 \ddot{\varphi}_1 L_1^2 + m_2 \ddot{\varphi}_1 L_1^2 + m_2 L_1 L_2(\ddot{\varphi}_2 cos(\varphi_2 - \varphi_1) - \dot{\varphi}_2 sin(\varphi_2 - \varphi_1) \cdot (\dot{\varphi}_2 - \dot{\varphi}_1))$$

$$\frac{\partial E_k}{\partial \varphi_1} = m_2 L_1 L_2 \dot{\varphi}_1 \dot{\varphi}_2 sin(\varphi_2 - \varphi_1)$$

$$E_p = -m_1 g L_1 cos\varphi_1 - m_2 g(L_1 cos\varphi_1 + L_2 cos\varphi_2)$$

$$\frac{\partial E_p}{\partial \varphi_1} = (m_1 + m_2) g L_1 sin\varphi_1$$

$$m_1 \ddot{\varphi}_1 L_1^2 + m_2 \ddot{\varphi}_1 L_1^2 + m_2 L_1 L_2(\ddot{\varphi}_2 cos(\varphi_2 - \varphi_1) - \dot{\varphi}_2 sin(\varphi_2 - \varphi_1) \cdot (\dot{\varphi}_2 - \dot{\varphi}_1)) - m_2 L_1 L_2 \dot{\varphi}_1 \dot{\varphi}_2 sin(\varphi_2 - \varphi_1) + (m_1 + m_2) g L_1 sin\varphi_1 = 0$$

2

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\varphi}_2}\right) - \frac{\partial E_k}{\partial \varphi_2} + \frac{\partial E_p}{\partial \varphi_2} = 0$$

$$E_k = \frac{m_1 \dot{\varphi}_1{}^2 L_1^2}{2} + \frac{m_2}{2}\left(\dot{\varphi}_1{}^2 L_1^2 + \dot{\varphi}_2{}^2 L_2^2 + 2\dot{\varphi}_1 L_1 \dot{\varphi}_2 L_2 cos(\varphi_2 - \varphi_1)\right)$$

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\varphi}_2}\right) = \frac{d}{dt}(m_2 \dot{\varphi}_2\, L_2^2 + m_2 L_1 \dot{\varphi}_1 L_2 cos(\varphi_2 - \varphi_1))$$

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\varphi}_2}\right) = m_1 \ddot{\varphi}_2\, L_2^2 + m_2 L_1 L_2(\ddot{\varphi}_1 cos(\varphi_2 - \varphi_1) - \dot{\varphi}_1 sin(\varphi_2 - \varphi_1) \cdot (\dot{\varphi}_2 - \dot{\varphi}_1))$$

$$\frac{\partial E_k}{\partial \varphi_2} = -m_2 L_1 L_2 \dot{\varphi}_1 \dot{\varphi}_2 sin(\varphi_2 - \varphi_1)$$

$$E_p = -m_1 g L_1 cos\varphi_1 - m_2 g(L_1 cos\varphi_1 + L_2 cos\varphi_2)$$

$$\frac{\partial E_p}{\partial \varphi_2} = m_2 g L_2 sin\varphi_2$$

$$m_1 \ddot{\varphi}_2\, L_2^2 + m_2 L_1 L_2(\ddot{\varphi}_1 cos(\varphi_2 - \varphi_1) - \dot{\varphi}_1 sin(\varphi_2 - \varphi_1) \cdot (\dot{\varphi}_2 - \dot{\varphi}_1)) + m_2 L_1 L_2 \dot{\varphi}_1 \dot{\varphi}_2 sin(\varphi_2 - \varphi_1) + m_2 g L_2 sin\varphi_2 = 0$$

Linearyzacja równań

$$m_1\ddot{\varphi}_1 L_1^2 + m_2\ddot{\varphi}_1 L_1^2 + m_2L_1L_2(\ddot{\varphi}_2 cos(\varphi_2 - \varphi_1) - \dot{\varphi}_2 sin(\varphi_2 - \varphi_1) \cdot (\dot{\varphi}_2 - \dot{\varphi}_1)) - m_2L_1L_2\dot{\varphi}_1\dot{\varphi}_2 sin(\varphi_2 - \varphi_1) + (m_1 + m_2)gL_1 sin\varphi_1 = 0$$

$$m_1\ddot{\varphi}_2 L_2^2 + m_2L_1L_2(\ddot{\varphi}_1 cos(\varphi_2 - \varphi_1) - \dot{\varphi}_1 sin(\varphi_2 - \varphi_1) \cdot (\dot{\varphi}_2 - \dot{\varphi}_1)) + m_2L_1L_2\dot{\varphi}_1\dot{\varphi}_2 sin(\varphi_2 - \varphi_1) + m_2gL_2 sin\varphi_2 = 0$$

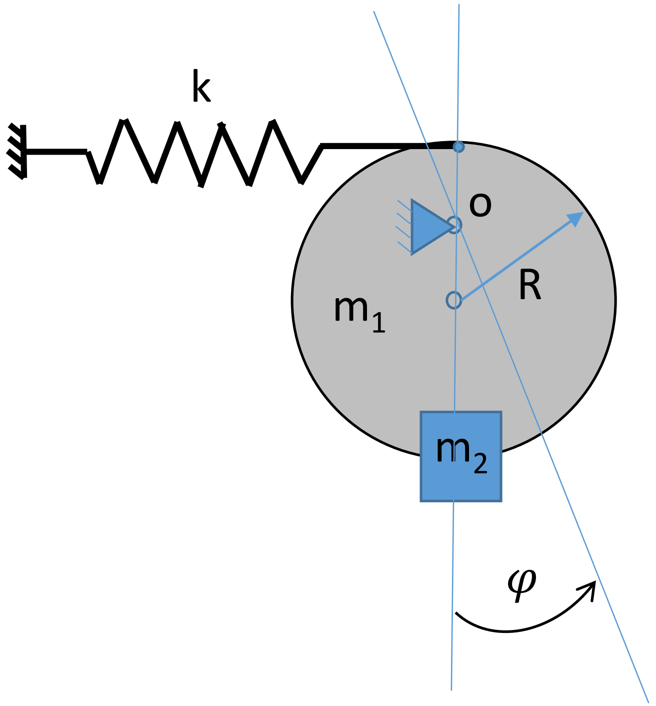
dla małych kątów: $sin\varphi = \varphi$ oraz $cos\varphi = 1$

małe wielkości wyższych rzędów $\dot{\varphi}_1\dot{\varphi}_2(\varphi_2 - \varphi_1) = 0$ $\dot{\varphi}_2(\varphi_2 - \varphi_1) \cdot (\dot{\varphi}_2 - \dot{\varphi}_1) = 0$

$$\dot{\varphi}_1 sin(\varphi_2 - \varphi_1) \cdot (\dot{\varphi}_2 - \dot{\varphi}_1) = 0$$

$$(m_1 + m_2)\ddot{\varphi}_1 L_1^2 + m_2L_1L_2\ddot{\varphi}_2 + (m_1 + m_2)gL_1\varphi_1 = 0$$

$$m_1\ddot{\varphi}_2 L_2^2 + m_2L_1L_2\ddot{\varphi}_1 + m_2gL_2\varphi_2 = 0$$



$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\varphi}}\right) - \frac{\partial E_k}{\partial \varphi} + \frac{\partial E_p}{\partial \varphi} = 0$$

$$E_k = \frac{I_o \dot{\varphi}^2}{2}$$

$$I_o = \frac{m_1 R^2}{2} + m_1\left(\frac{R}{2}\right)^2 + m_2\left(\frac{3}{2}R\right)^2$$

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\varphi}}\right) = \frac{d}{dt}(I_o \dot{\varphi}) = I_o \ddot{\varphi}$$

$$\frac{\partial E_k}{\partial \varphi} = 0$$

$$\frac{\partial E_p}{\partial \varphi} = m_2 \mathrm{g}\frac{3}{2}\mathrm{R}\sin\varphi + m_1 \mathrm{g}\frac{1}{2}\mathrm{R}\sin\varphi + \frac{kR^2}{4}\varphi$$

$$E_p = -m_2 \mathrm{g}\frac{3}{2}\mathrm{R}\cos\varphi - m_1 \mathrm{g}\frac{1}{2}\mathrm{R}\cos\varphi + \frac{k\left(\frac{R}{2}\varphi\right)^2}{2}$$

dla małych drgań

$\sin\varphi = \varphi$

$$I_o \ddot{\varphi} + m_2 \mathrm{g}\frac{3}{2}\mathrm{R}\sin\varphi + m_1 \mathrm{g}\frac{1}{2}\mathrm{R}\sin\varphi + \frac{kR^2}{4}\varphi = 0$$

$$\ddot{\varphi} + \varphi\frac{6m_2\mathrm{gR} + m_1\mathrm{g}2\mathrm{R} + kR^2}{3m_1R^2 + m_2(3R)^2} = 0$$

$$\omega = \sqrt{\frac{6m_2\mathrm{gR} + m_1\mathrm{g}2\mathrm{R} + kR^2}{3m_1R^2 + m_2(3R)^2}}$$