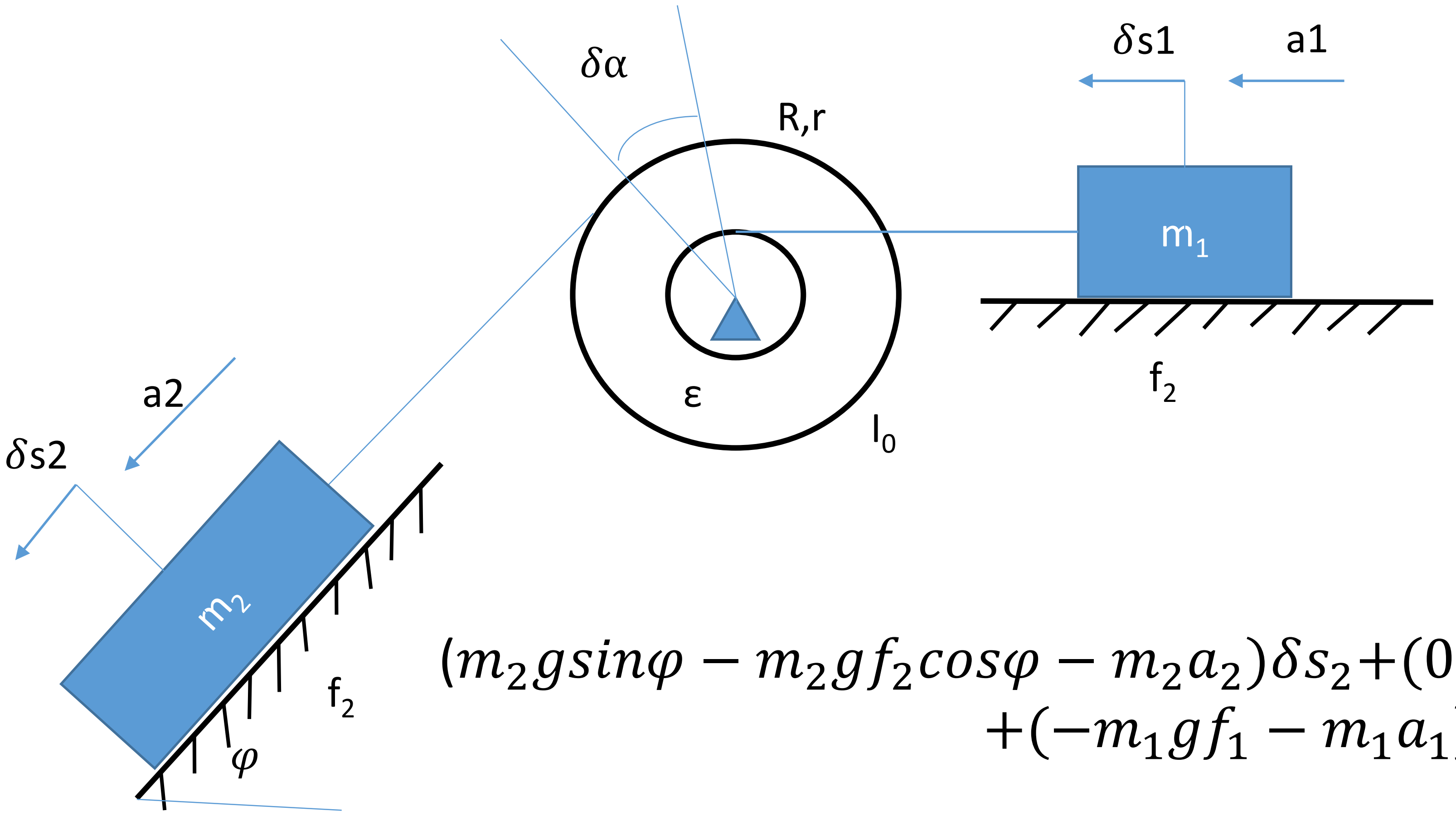


Z zasady prac przygotowanych wyznaczyć przyspieszenie

$$(m_2 g sin\varphi - m_2 g f_2 cos\varphi - m_2 a_2)\delta s_2 + (0 - \varepsilon I_0)\delta\alpha +$$
$$+(-m_1 g f_1 - m_1 a_1)\delta s_1 = 0$$

$\delta\alpha R = \delta s_2$ $\quad$ $\delta\alpha r = \delta s_1$

$a_2 = \varepsilon R$ $\quad$ $a_1 = \varepsilon r$

wszystko uzależniamy od przyspieszenia kątowego

$$(m_2 g sin\varphi - m_2 g f_2 cos\varphi - m_2 \varepsilon R)\delta\alpha R + (0 - \varepsilon I_0)\delta\alpha + (-m_1 g f_1 - m_1 \varepsilon r)\delta\alpha r = 0$$

$\delta\alpha$ wyciąmy przed nawias

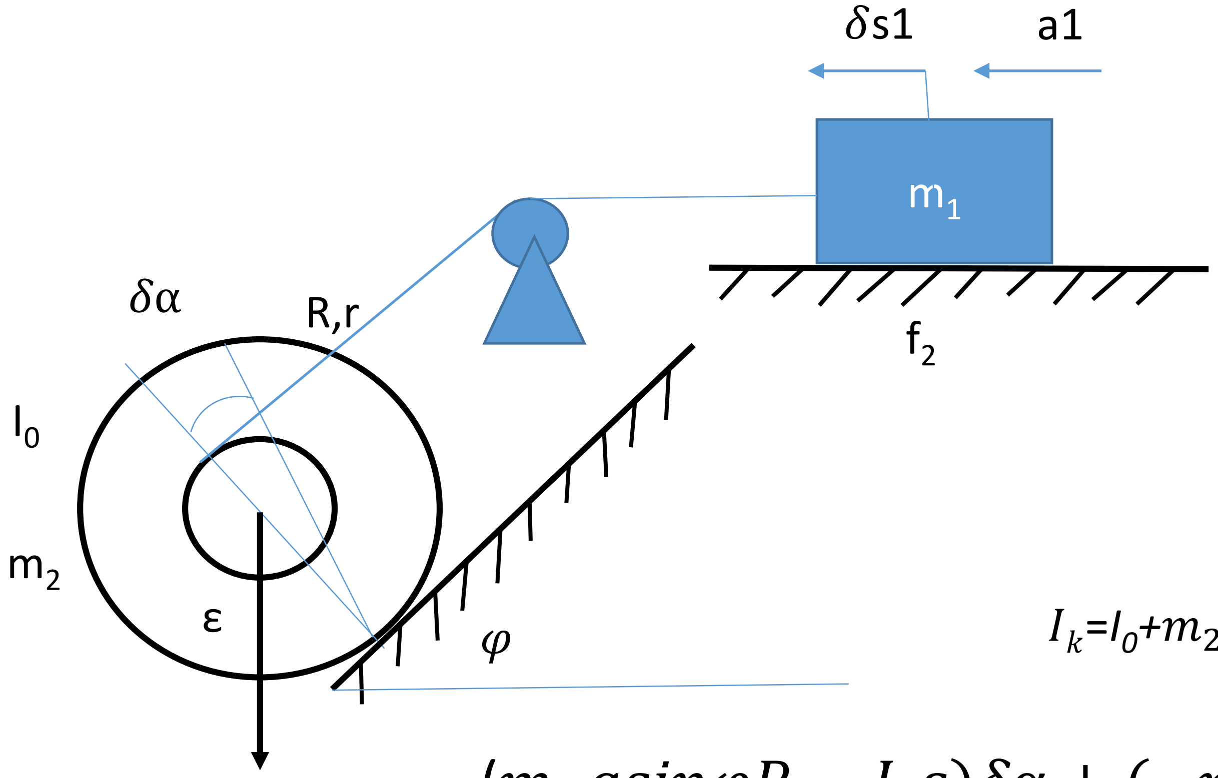
$$((m_2 g sin\varphi - m_2 g f_2 cos\varphi - m_2 \varepsilon R)R + (0 - \varepsilon I_0) + (-m_1 g f_1 - m_1 \varepsilon r)r)\delta\alpha = 0$$

ma być prawdziwe dla dowolnego $\delta\alpha$

$$R m_2 g sin\varphi - R m_2 g f_2 cos\varphi - R^2 m_2 \varepsilon - \varepsilon I_0 - r m_1 g f_1 - m_1 \varepsilon r^2 r = 0$$

$$\varepsilon(R^2 m_2 + I_0 + m_1 r^2) = R m_2 g sin\varphi - R m_2 g f_2 cos\varphi - r m_1 g f_1$$

$$\varepsilon = \frac{R m_2 g sin\varphi - R m_2 g f_2 cos\varphi - r m_1 g f_1}{R^2 m_2 + I_0 + m_1 r^2}$$



$I_k = I_0 + m_2 R^2$

$$(m_2 g sin\varphi R - I_k \varepsilon)\delta\alpha + (-m_1 g f_1 - m_1 a_1)\delta s_1 = 0$$

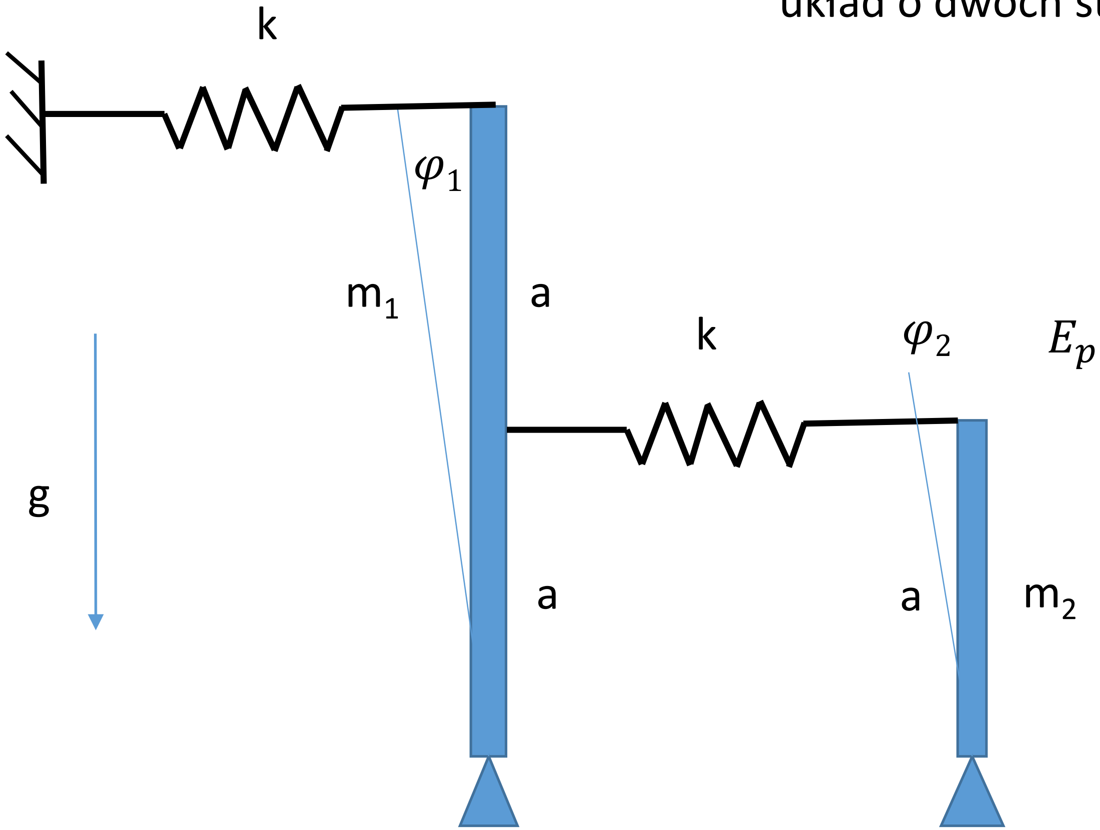
$\delta\alpha(\mathrm{R} + \mathrm{r}) = \delta s_1$

$\varepsilon(R + r) = a_1$

$$(m_2 g sin\varphi R - I_k \varepsilon)\delta\alpha + (-m_1 g f_1 - m_1 \varepsilon(R + r))\delta\alpha(\mathrm{R} + \mathrm{r}) = 0$$

$$\delta\alpha(m_2 g sin\varphi R - I_k \varepsilon - m_1 g f_1 (R + r) - m_1 \varepsilon (R + r)^{2}) = 0$$

$$\varepsilon = \frac{m_2 g sin\varphi R - m_1 g f_1 (R + r)}{m_1 (R + r)^{2} + I_0 + m_2 R^2}$$



układ o dwóch stopniach

$$E_k = \frac{I\,\omega^2}{2}$$

$$E_k = \frac{m_1 (2a)^{\,2} (\dot{\varphi}_1)^{\,2}}{2 \cdot 3} + \frac{m_2 a^{\,2} (\dot{\varphi}_2)^{\,2}}{2 \cdot 3}$$

$$E_p = m_1 g\ a\ \cos\varphi_1 + 0.5 m_2 g\ a\ \cos\varphi_2 + 0.5k((\varphi_1 - \varphi_2)a)^2 + 0.5k(2\varphi_1 a)^2$$

środek ciężkości

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\varphi}_1}\right) - \frac{\partial\, E_k}{\partial \varphi_1} + \frac{\partial\, E_p}{\partial \varphi_1} = 0$$

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\varphi}_2}\right) - \frac{\partial\, E_k}{\partial \varphi_2} + \frac{\partial\, E_p}{\partial \varphi_2} = 0$$

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\varphi}_1}\right) - \frac{\partial\, E_k}{\partial \varphi_1} + \frac{\partial\, E_p}{\partial \varphi_1} = 0$$

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\varphi}_1}\right) = \frac{d}{dt}\left(\frac{m_1(2a)^{\,2}\dot{\varphi}_1}{3}\right) = \frac{m_1(2a)^{\,2}\ddot{\varphi}_1}{3}$$

$$\frac{\partial\, E_k}{\partial \varphi_1} = 0$$

$$\frac{\partial\, E_p}{\partial \varphi_1} = -m_1 g\ a\ \sin\varphi_1 + \mathrm{k}a^2\ (\varphi_1 - \varphi_2) + 4\mathrm{k}a^2\varphi_1$$

$$\frac{m_1(2a)^{\,2}\ddot{\varphi}_1}{3} - m_1\ g\ a\ \sin\varphi_1 + \mathrm{k}a^2\ (\varphi_1 - \varphi_2) + 4\mathrm{k}a^2\varphi_1 = 0$$

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\varphi}_2}\right) - \frac{\partial\, E_k}{\partial \varphi_2} + \frac{\partial\, E_p}{\partial \varphi_2} = 0$$

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\varphi}_2}\right) = \frac{d}{dt}\left(\frac{m_2 a^{\,2} \dot{\varphi}_2}{3}\right) = \frac{m_2 a^{\,2} \ddot{\varphi}_2}{3}$$

$$\frac{\partial\, E_k}{\partial \varphi_2} = 0$$

$$\frac{\partial\, E_p}{\partial \varphi_2} = -0.5 m_2 \mathrm{g\ a\ sin}\varphi_2 - \mathrm{k}\ a^2(\varphi_1 - \varphi_2)$$

$$\frac{m_2 a^{\,2} \ddot{\varphi}_2}{3} - 0.5 m_2 \mathrm{g\ a\ sin}\varphi_2 - \mathrm{k}\ a^2(\varphi_1 - \varphi_2) = 0$$

$$\frac{m_1(2a)^2\ddot{\varphi}_1}{3} - m_1 \text{ g a } \sin\varphi_1 + \mathrm{k}a^2(\varphi_1 - \varphi_2) + 4\mathrm{k}a^2\varphi_1 = 0$$

$$\frac{m_2 a^2\ddot{\varphi}_2}{3} - 0.5m_2\text{g a } \sin\varphi_2 - \mathrm{k}\, a^2(\varphi_1 - \varphi_2) = 0$$

dla małych drgań $\sin\varphi = \varphi$

$$\frac{m_1(2a)^2\ddot{\varphi}_1}{3} - m_1 \text{ g a } \varphi_1 + \mathrm{k}a^2(\varphi_1 - \varphi_2) + 4\mathrm{k}a^2\varphi_1 = 0$$

$$\frac{m_2 a^2\ddot{\varphi}_2}{3} - 0.5m_2\text{g a } \varphi_2 - \mathrm{k}\, a^2(\varphi_1 - \varphi_2) = 0$$

$$\begin{bmatrix} \frac{m_1(2a)^2}{3} & 0 \\ 0 & \frac{m_2 a^2}{3} \end{bmatrix} \begin{bmatrix} \ddot{\varphi}_1 \\ \ddot{\varphi}_2 \end{bmatrix} + \begin{bmatrix} -m_1\text{g a} + 5\mathrm{k}a^2 & -\mathrm{k}a^2 \\ -\mathrm{k}a^2 & -0.5m_2\text{g a} + \mathrm{k}\, a^2 \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = 0$$

$$\boldsymbol{det}\left(\boldsymbol{K}-\omega_{\mathbf{0}}^{\mathbf{2}}\boldsymbol{M}\right)=\boldsymbol{det}(\boldsymbol{K}-\lambda\boldsymbol{M})=0$$

$$det\begin{bmatrix}-m_1\text{g a}+5\text{k}a^2-\lambda\frac{m_1(2a)^{\,2}}{3} & -\text{k}a^2\\ -\text{k}a^2 & -0.5m_2\text{g a}+\text{k}\,a^2-\lambda\frac{m_2a^{\,2}}{3}\end{bmatrix}=0$$

dane: $m_1=2kg, m_2=1, kg\ k=1000\frac{N}{m}, \boldsymbol{a}=0.2m$

$$det\begin{bmatrix}196.1-\lambda\cdot 0.1 & -40\\ -40 & 39-\lambda\cdot 0.01\end{bmatrix}=0$$

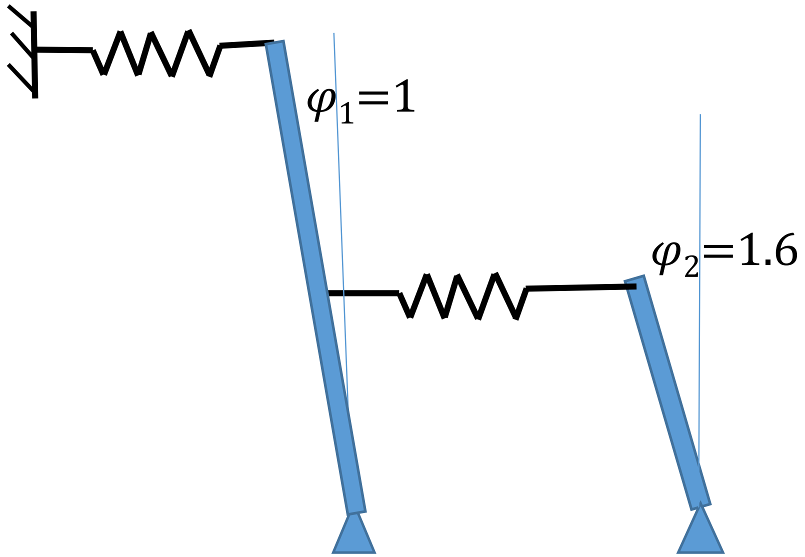
$$\lambda^2 0.001-\lambda\cdot 5.9+6048=0$$

$\omega=\sqrt{\lambda}$ $\qquad$ $\omega_1=36.34, \omega_2=67.7$

dla ω_1 $$\begin{bmatrix} 196.1 - \lambda \cdot 0.1 & -40 \\ -40 & 39 - \lambda \cdot 0.01 \end{bmatrix} \begin{bmatrix} q_{011} \\ q_{012} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 64 & -40 \\ -40 & 25.8 \end{bmatrix} \begin{bmatrix} q_{011} \\ q_{012} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Dla $q_{011} = 1$ wynosi $q_{012} = 1.6$



dla ω_2 $$\begin{bmatrix} 196.1 - \lambda \cdot 0.1 & -40 \\ -40 & 39 - \lambda \cdot 0.01 \end{bmatrix} \begin{bmatrix} q_{021} \\ q_{022} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -262,2 & -40 \\ -40 & -6.8 \end{bmatrix} \begin{bmatrix} q_{021} \\ q_{022} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Dla $q_{021} = 1$ wynosi $q_{022} = -6.2$